This is a solution to the second question featured in xkcd comic #135:
http://xkcd.com/135/

The text is as follows:

You are at the center of a 20m equilateral triangle with a raptor at each corner. The top raptor has a wounded leg and is limited to a top speed of 10 m/s.

The raptors will run toward you. At what angle should you run to maximize the time you stay alive?

[From question #1, we know a healthy raptor has a top speed of 25 m/s, accelerates at 4 m/s/s, and you quickly accelerate to your top speed of 6 m/s.]

The text implies that the raptors will continuously rotate so that they're always facing towards you, resulting in a curved trajectory. Other solutions online use this approach to write a program to calculate your lifetime vs. angle.

In this solution, I will make the assumption that the raptors are intelligent enough to move in such a way as to intercept you in the least time possible – that is, in a straight line. This doesn't hold true to a strict interpretation of the question, but I think it's an interesting question in its own right.

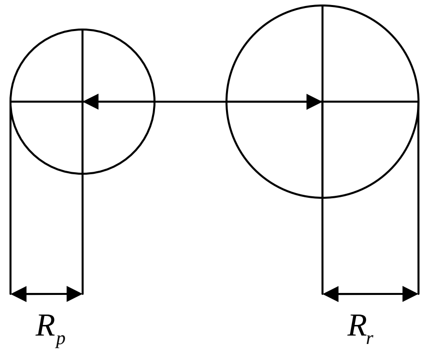
Suppose you only face 1 raptor. At some particular time after the running begins, you and the raptor will be a particular distance away from your original positions. Since you are each free to choose the angle of departure, we can represent all possible positions at a particular instance with a pair of circles centered on the starting positions. The radii are determined by the velocities, accelerations, and time:

$$R_p = v_p t$$

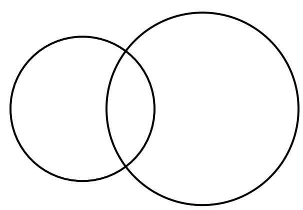
$$R_r = \begin{cases} \frac{1}{2} a_r t^2 & t < \frac{v_r}{a_r} \\ \frac{v_r^2}{2a_r} + v_r \left(t - \frac{v_r}{a_r} \right) & t > \frac{v_r}{a_r} \end{cases}$$

The p subscripts refer to the person; r refers to the raptor. The piecewise formula for R_r reflects the fact that there are two phases to the raptor's motion – acceleration and constant velocity.

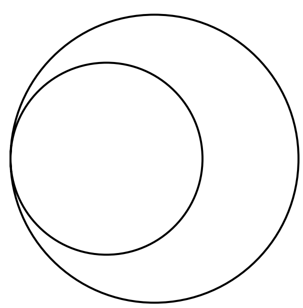
Graphically, we can represent the motion like so:



These circles grow as a function of time. Eventually, they will barely touch, and then they will overlap (forming what looks like a Venn diagram):



Eventually, the raptor's circle will completely subsume your circle:

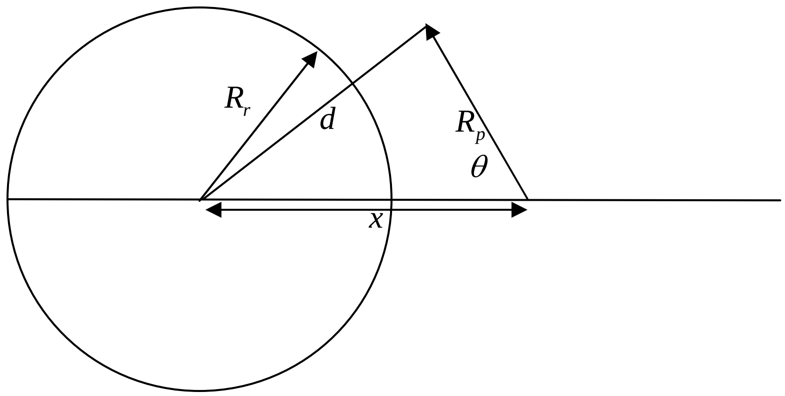


Remember that we don't control the rate of growth of the circles, or their initial spacing.

The only thing we control is the angle at which you run, which translates into your position on the circle as it grows. If you decide to run directly at the raptor, you've chosen to die as quickly as possible. This corresponds to when the two circles just barely touch.

If you choose some other angle, you'll survive until the raptor's circle grows to touch your chosen position on the circle. To live the longest, you want to run straight away- then the raptor won't catch you until its circle has entirely covered yours.

Hopefully that gets the idea cross. The goal now is to determine how long it takes to be caught for a particular angle- that is, time as a function of angle. We'll begin with a diagram (I've omitted the circle of the person for clarity):



The distance *x* is the horizontal distance between the original position of the raptor and person. The distance *d* is the distance between the position of the person and the original position of the raptor. When *d* equals R_r, the person is caught.

We need to express *d* as a function of time. I'll impose a Cartesian coordinate system with the $\hat{i}$ pointing left and $\hat{j}$ pointing up. Then:

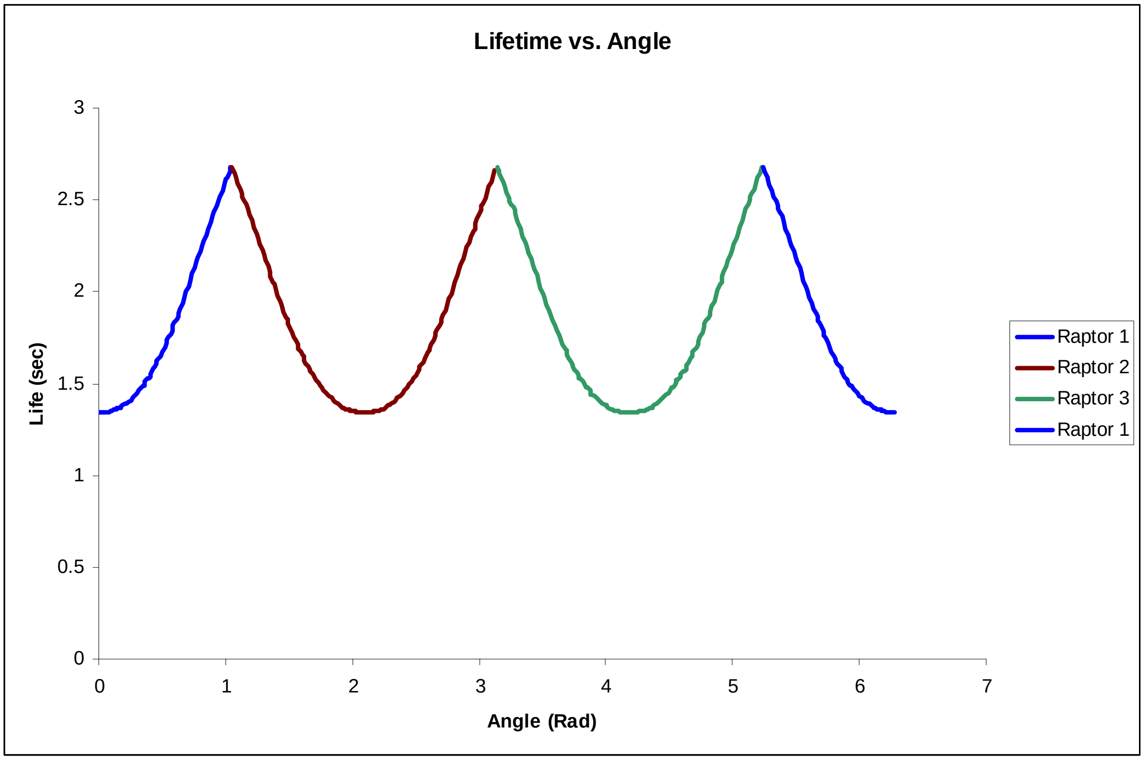
$$d = \left[\left(v_p \cos(\theta) t - x\right)^2 + \left(v_p \sin(\theta) t\right)^2\right]^{1/2}$$

This is pretty much it, then – we calculate R_r according to the equation we stated near the beginning (being sure to account for its piecewise nature), and compare it to *d*. Once they're equal, the person is devoured. Graphically, the raptor's circle has grown to a size such that it overlaps your position.

Note that we're assuming that the raptor is intelligent enough to run directly towards this point (again, counter to the wording of the question).

Now to the specifics of this question. Doing some geometry, we find that the initial distance to each raptor is $20/\sqrt{3}\,\text{m}$. We call the angle relative to the top raptor's position θ_1, which forces us to call $\theta_2 = \theta_1 + \frac{2\pi}{3}$ and $\theta_3 = \theta_1 + \frac{4\pi}{3}$.

All of the work was done on the accompanying excel spreadsheet. We graph the lifetime vs. angle:



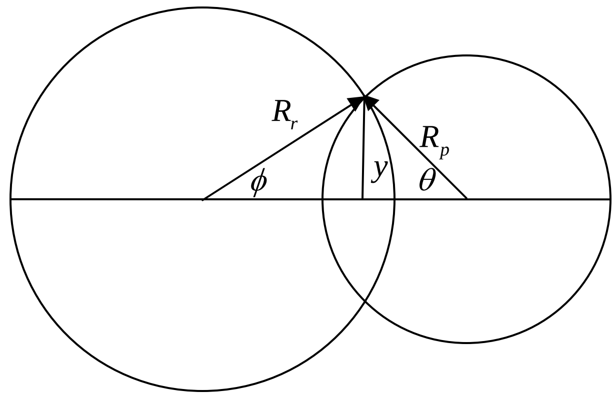
The maximum lifetime of 2.68 seconds occurs when you run directly between any two of the three raptors. The minimum of 1.34 occurs when you run directly at any of the 3 raptors.

You may be surprised to find that the wounded raptor makes no difference. It takes 2.5 seconds for it to accelerate to its maximum speed; until then, it behaves exactly like the unwounded raptors (which accelerate for 6.25 seconds). The maximum lifetime of the person is 2.68 seconds, which means that the fact that one raptor is wounded only plays a role in the calculation for the last .14 seconds of the motion (at most). Given the precision of my calculations, this simply didn't play a role in the results.

What if we made things easier and assumed that each raptor instantly accelerated to its top speed? The procedure is similar, except we can get away with solving for the time directly:

$$t = \frac{d}{v_x} = \frac{x}{v_r \cos(\phi) + v_p \cos(\theta)}$$

Where x is again the initial separation between the person and raptor, and ϕ is defined below:

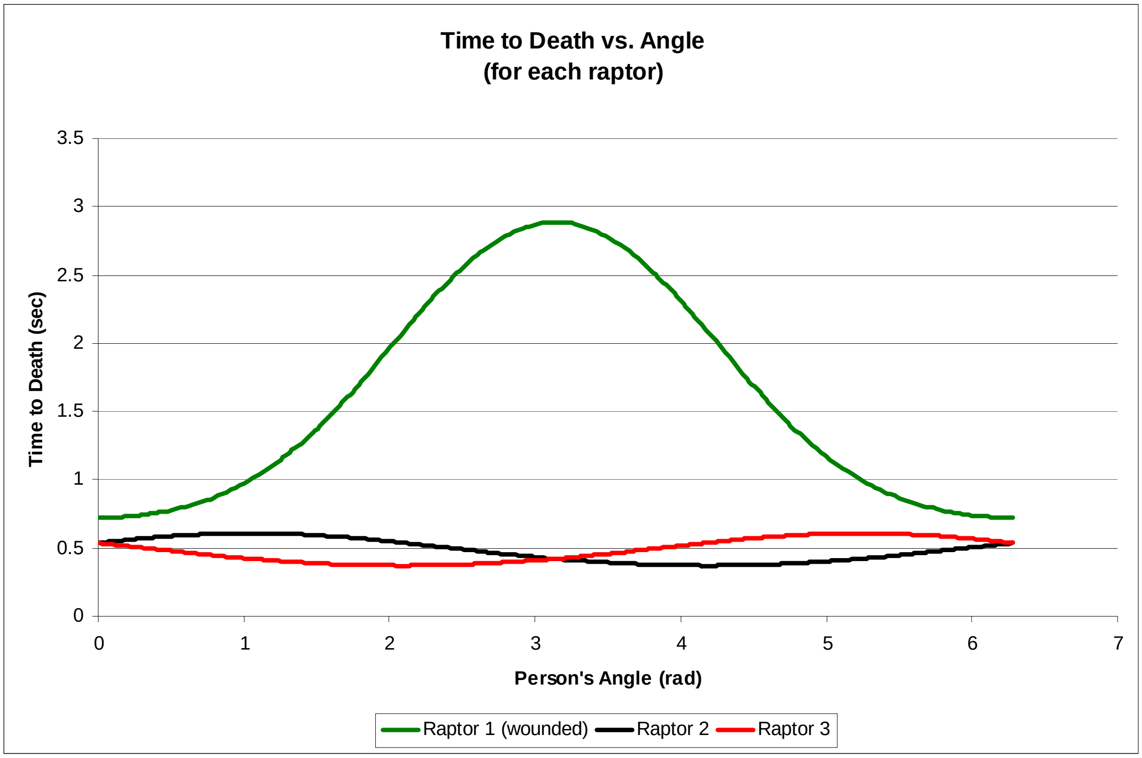


We can determine ϕ as a function of θ since they are components of two right triangles which share a common side, *y*. Specifically:

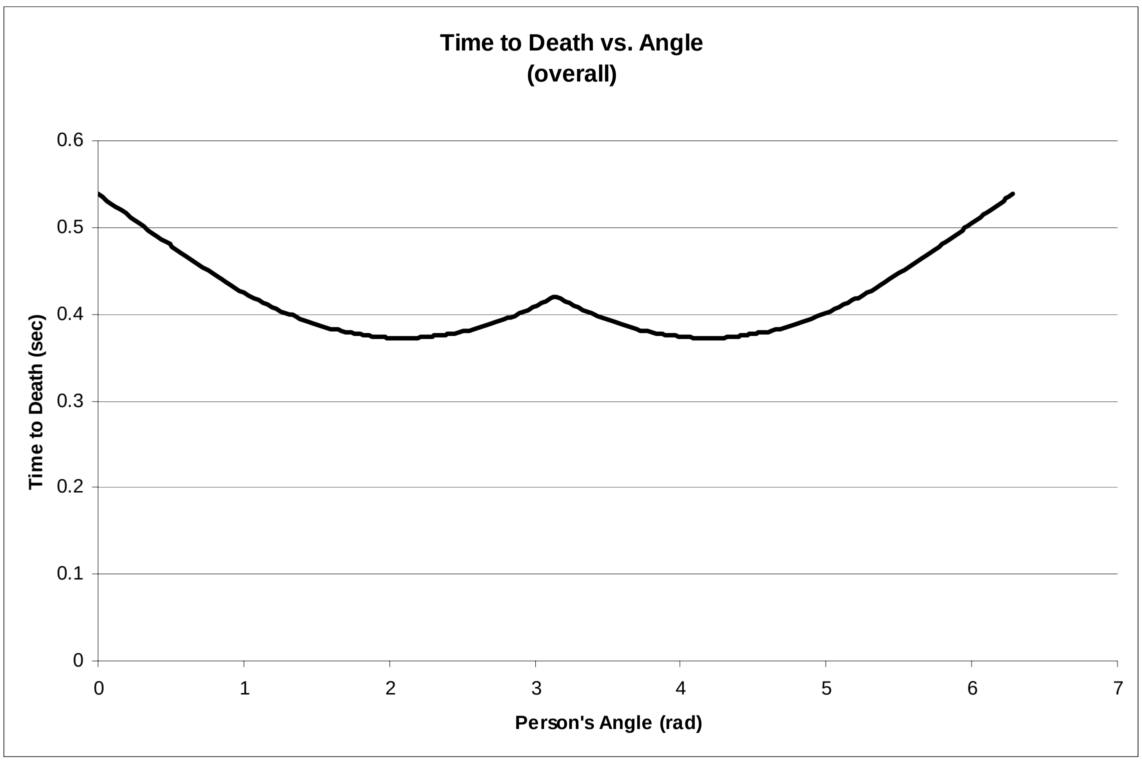
$$\phi = \sin^{-1}\left(\frac{v_p}{v_r}\sin(\theta)\right)$$

You'll note that the equation for time involves only the x component of velocity – since we've defined ϕ this way, we know that they'll intersect when they meet in the horizontal dimension (or the vertical, for that matter – but we don't know the value of *y* explicitly).

This, again, is pretty much it (we still need to make sure we define separate values of θ and ϕ, of course). We plot the time it takes for each raptor to reach you as a function of angle (0 is towards the wounded raptor, as before):



We see that even if you run directly at the wounded raptor, the other raptors will catch you first! Plotting lifetime vs. angle gives:



The minimum lifetime is .37 seconds and the maximum is .54 seconds.

Note:
I’ve made my excel spreadsheet available so that you may play around with it if you like. Note that in order to calculate the results for the case which includes acceleration, I wrote a macro. I deleted it from the uploaded copy since macros are capable of harming your computer, and while I certainly wouldn’t write such a macro, I don’t want to make anyone nervous. You can still manually change theta to particular values to see the results.

Enjoy!

Colin Campbell
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