Draft 3

Candidacy Exam
Department of Physics
January 17, 2004

Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and the approximations required to arrive at your solution. If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate. Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all the *four* problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants:

Avogadro's number	N_A	$6.023 \times 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A}^{-2}$
Gravitational constant	G	$6.673 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{s}^{-2}\,\mathrm{kg}^{-1}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m}^{-2}\,\mathrm{K}^{-4}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$

I–1 A ball of mass M and radius R is thrown onto a flat horizontal surface. The coefficient of kinetic friction between the ball and the surface is μ, and the moment of inertia of the ball about its center of mass is I. Initially the ball is moving with speed v_0 and is not rotating.

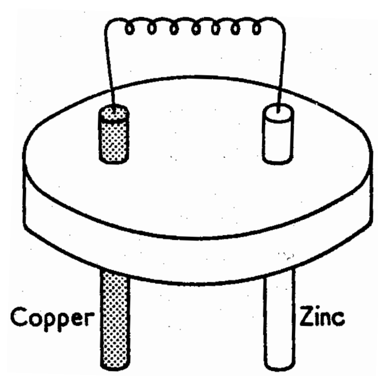
(a) Find an expression for the time t at which pure rolling motion begins.

(b) Determine the ratio of the speed when pure rolling begins to the initial speed v_0.

(c) For the case that the ball has a uniform spherical distribution of mass, derive its moment of inertia about its center.

I–2 Explain the following observations:

(a) When a solenoid of copper wire is joined to rods of copper and zinc, and fixed to a cork, as in the figure, and the whole device floats on the surface of dilute sulphuric acid, the axis of the solenoid points in a definite direction.



(b) A light aluminum ring, suspended from threads with its plane vertical, swings away from your hand while you are bringing one pole of a bar magnet towards it, and towards your hand while you are withdrawing the magnet.

I–3 A particular thermodynamical system is geometrically a sphere of area A, which is the only independent macroscopic parameter needed to specify the thermodynamic state of the system. The energy and temperature are related to A by

$$A = \alpha E^2, \qquad ET = \gamma, \tag{1}$$

where α and γ are constants.

(a) The sphere radiates as a black body. Find an equation for the rate of change of its area $A(t)$ as a function of time t. Show that the system disappears after a finite time, i.e., that its area goes to zero. Find how long it takes for the system to disappear for a given initial area $A(0)$.

(b) Find the entropy of the system as a function of A, given that $S(A = 0) = 0$.

NOTE: The system you have just solved corresponds to what is thought to be the fundamental description of a black hole of area A in quantum gravity and black hole thermodynamics.

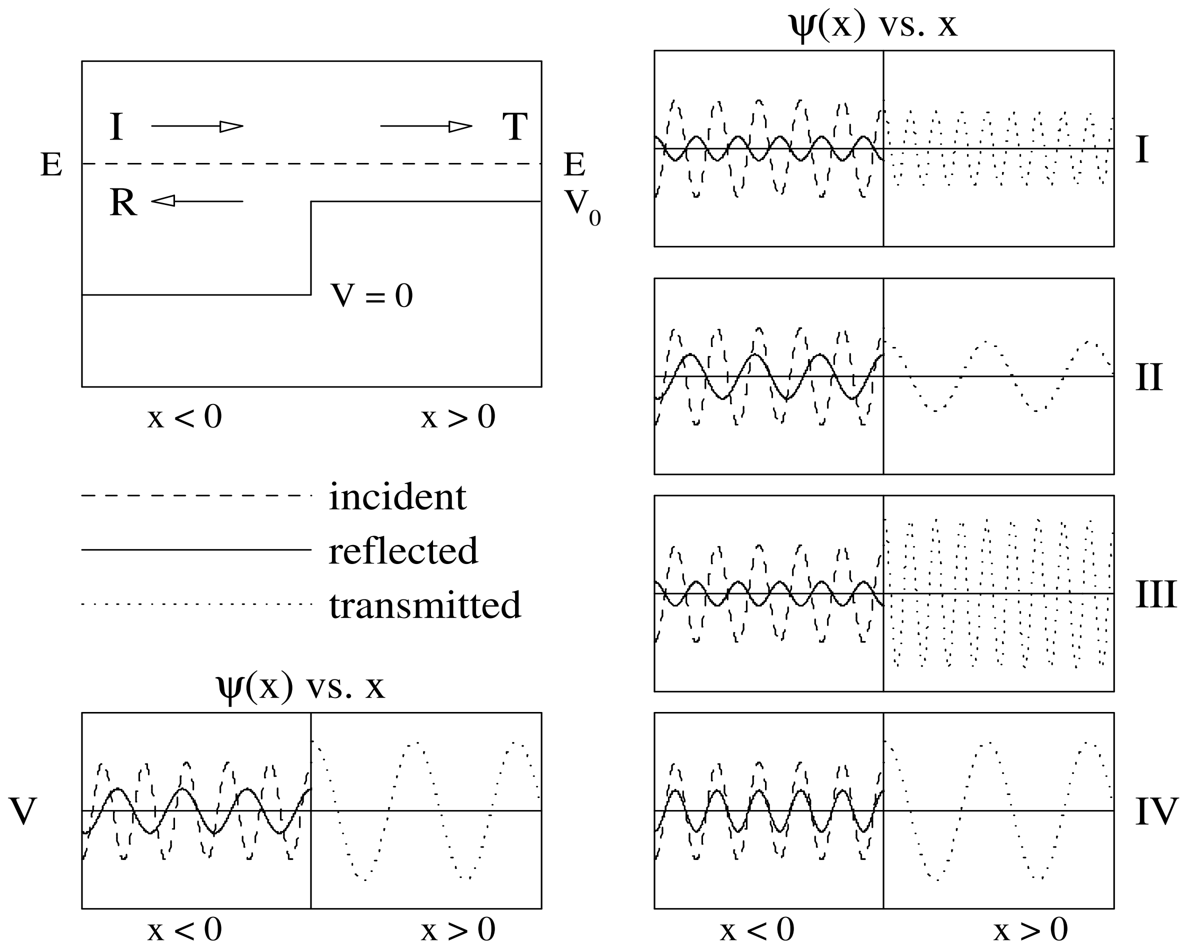
I–4 A beam of particles of energy E is incident, from the left, on a step potential of height $V_0 < E$ as shown in the figure on the next page. This problem is often analyzed using plane-wave solutions of the time-independent Schrödinger equation with incident (I), reflected (R) and transmitted (T) components represented by the solution:

$$\psi(x) = \begin{cases} Ie^{ikx} + Re^{-ikx} & \text{for } x \leq 0 \\ Te^{iqx} & \text{for } 0 \leq x \end{cases}$$

where $k = \sqrt{2mE/\hbar^2}$ and $q = \sqrt{2m(E - V_0)/\hbar^2}$.

Which of the waveforms shown in the figure on the next page (namely I, II, III, IV, or V) best represents the true solution in this approach? Each plot I, II, III, IV, and V shows a possible solutions to the scattering problem illustrated in the upper left corner. The curves for incident (dashed), reflected (solid) and transmitted (dotted) plane wave solutions represent the real part of the wave function. ***Note that the plotting software has produced slightly inaccurate plots at $x = 0$: (i) the curves should have zero gradient there, and (ii) the ends of the dotted and dashed lines may be slightly displaced vertically by about a mm.***

Explain your answer. Give a reason for each wrong plot being wrong.


ψ(x) vs. x
I
T
E
E
R
V_0
V = 0
x < 0
x > 0
incident
reflected
transmitted
ψ(x) vs. x
V
I
II
III
IV
x < 0
x > 0
x < 0
x > 0

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Part II

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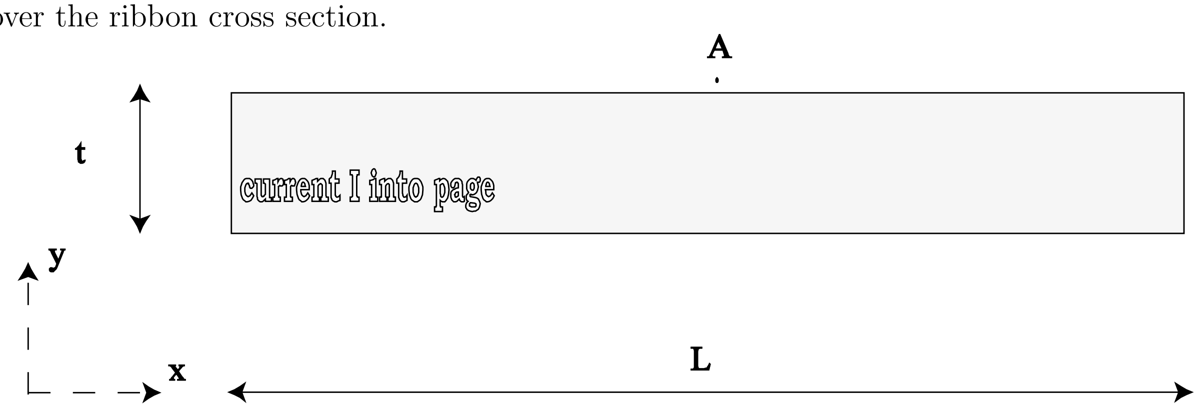
II–1 In *classical* mechanics, a model of a linear molecule of three atoms consists of a central atom of mass M, and two end atoms of mass m. Springs of constant K connect the central atom to the two ends. All the atoms lie on a single line.

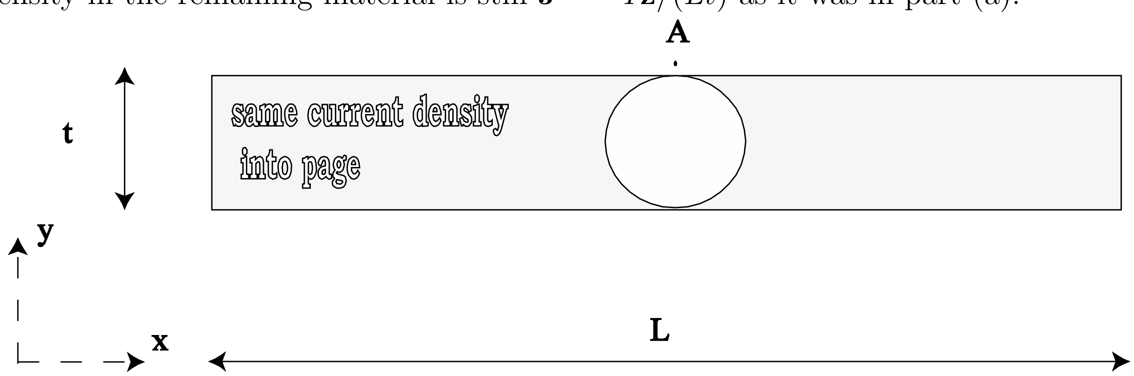
Derive the frequencies of vibrational motion along the axis of the molecule. Sketch the normal modes for each frequency.

II–2 Assume that a thin ribbon of wire, with thickness t and width L carries a total current I. (Assume $t \ll L$.) The following picture shows the cross section of the ribbon of wire. The current I flows along the negative z axis, into the page in the current density is constant over the ribbon cross section.



(a) Determine the magnitude and direction of the magnetic field at point A, right above the center of the ribbon and a very small distance from the top surface.

(b) Now a similar ribbon is modified by removing the conductor in a cylindrical region around the longitudinal axis of the ribbon. The diameter of the hole is t. The current density in the remaining material is still $\mathbf{J} = -I\hat{\mathbf{z}}/(Lt)$ as it was in part (a):



Determine the magnitude and direction of the magnetic field at point A, just above the hole.

II–3 A 20 g icecube at temperature $-15\,^{\circ}\mathrm{C}$ is placed in a pond whose temperature is $+10\,^{\circ}\mathrm{C}$.

(a) Calculate the change in entropy of the system as the ice cube comes into equilibrium with the pond. [The latent heat of fusion for water is 334 J/g, the specific heat of water is $4.2\,\mathrm{J/(g\cdot K)}$, the specific heat of ice is $2.1\,\mathrm{J/(g\cdot K)}$, and $0\,^{\circ}\mathrm{C} = 273\,\mathrm{K}$.]

(b) What sign would you expect for this change in entropy and why?

II–4 A particle of mass m moves in a one-dimensional square well potential with walls of infinite potential a distance L apart.

(a) What are the energy eigenvalues and eigenstates for the ground state and the first excited state of this system?

Let these two states be $|\psi_1\rangle$ and $|\psi_2\rangle$, and at time $t = 0$, suppose the state is $(|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$.

(b) At a given later time, what is the expectation value of the energy of the particle occupying a state which is a linear combination of equal amounts of these two lowest energy eigenstates?

(c) With the same initial condition, what is the probability, at a given later time, that the particle will be in the right-hand half of the well?

(d) Is there any ambiguity in the answers?