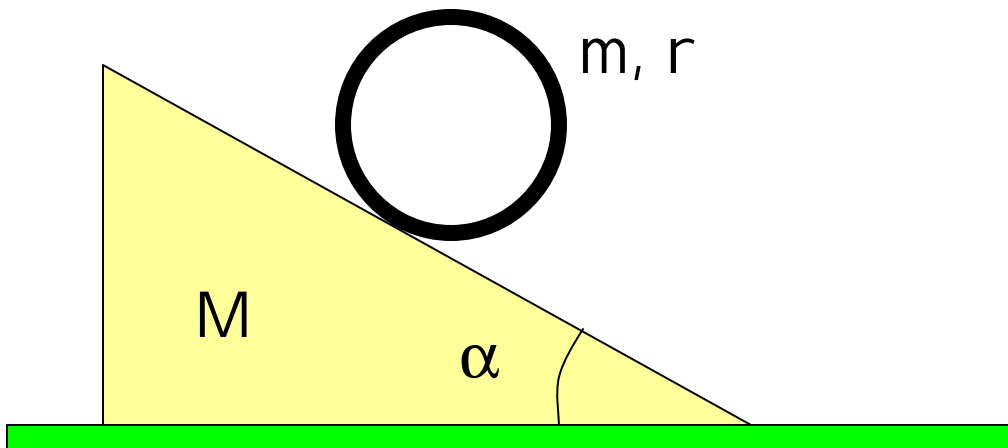


Candidacy Exam Spring 2003

Problem 1

A hoop of mass m and radius r rolls without slipping down a wedge of mass M , which has a fixed angle α with the horizontal. The wedge is on a frictionless horizontal surface, and can slide.

- Select appropriate coordinates, and write down the Lagrange equations for the motion of this system.
- Using any formalism you like, determine an expression for the position and velocity of the wedge as a function of time.



Problem 2

Consider a negatively charged muon moving in the Coulomb field of a nucleus of radius R and charge Z . Assuming that the nuclear charge Z is distributed evenly inside a sphere of radius R , find the lowest order correction for the energy of the ground state relative to the case of a point-like nucleus.

The groundstate wavefunction of a hydrogen-like atom is given by

$$\Psi(r) = \frac{a^{-\frac{3}{2}}}{\sqrt{\pi}} e^{-\frac{r}{a}} \quad \text{where } a \text{ is the Bohr radius: } a = \frac{e_0 h^2}{\mu m e^2}$$

In the calculation assume that the Bohr radius of the system is much larger than R , and the mass of the nucleus is much larger than the muon mass.

(The muon charge is equal to that on an electron, and the mass of the muon is 207 times the mass of the electron.)

Problem 3

A sphere of radius a is uniformly charged with density ρ . The sphere rotates with a constant angular velocity ω about the z -axis. Find an expression for the magnetic field induced at the center of the sphere.

Problem 4

Consider an ideal gas (of 1 mole) which satisfies $PV=RT$; the internal energy U depends on T and is independent of P and V .

- (a) Starting from the first law, show that

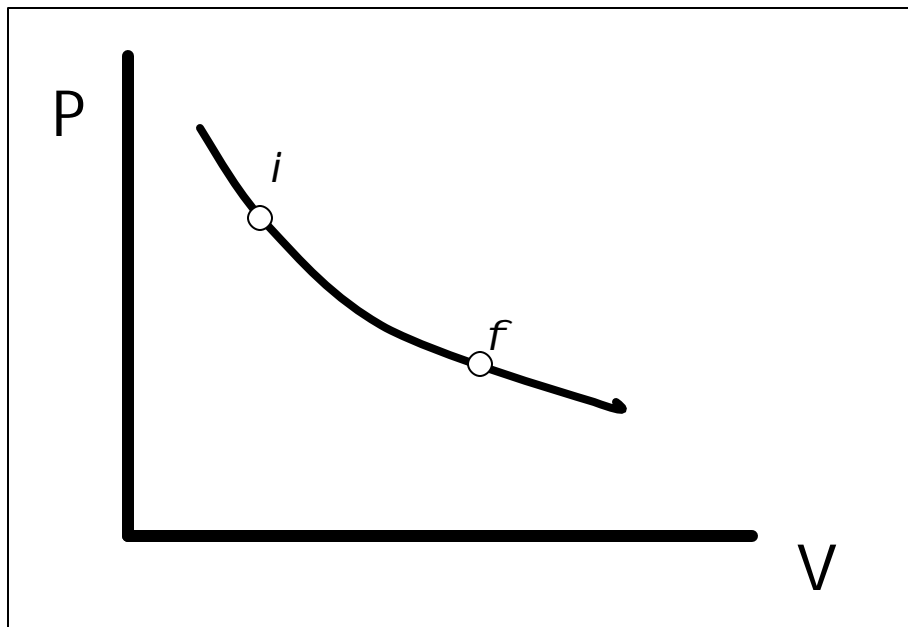
$$C_P - C_V = R$$

- (b) Show that for a quasistatic adiabatic process

$$PV^\gamma = \text{constant}$$

where $\gamma = C_P/C_V$ (assume constant specific heat)

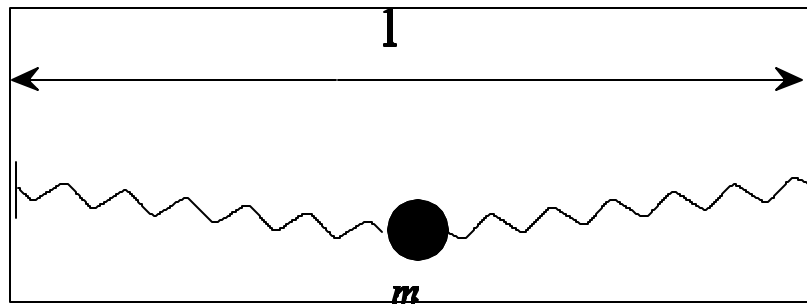
- c) A sketch of the P - V diagram of an isothermal expansion from initial state i to final state f is shown. Sketch the curve of an adiabatic process starting from the same initial state. Does the temperature rise or fall in the process?



Problem 5

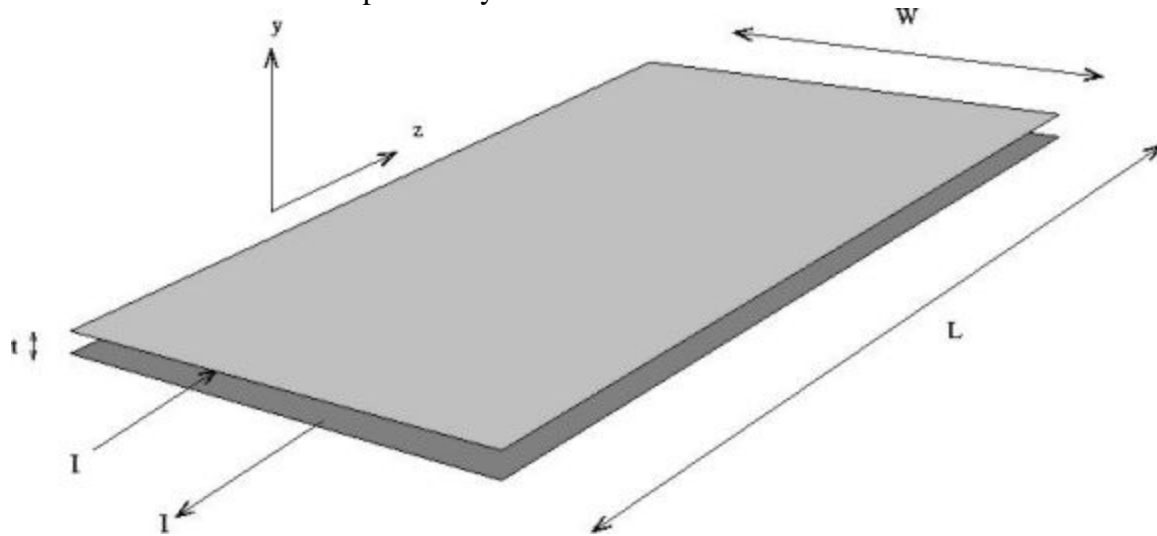
A particle of mass m is suspended from two identical springs as depicted in the figure below. The Hooke's Law constant for the springs is k and the length of the springs in the equilibrium is l_s . The distance between the supports is $l > 2l_s$.

- (a) Find the equilibrium position of the system.
- (b) Calculate the frequencies of the oscillations assuming that the particle can move only horizontally and vertically (*i.e.* in the plane of the paper) and that the mass of the particle is small enough so that the equilibrium angle between the springs and the horizontal is very small.



Problem 6

A velocity selector is fabricated from two rectangular conducting sheets, each of length L and width W . The sheets are separated by distance t .



The conductor sheets are oriented as shown in the figure, one in the plane $y = 0$ and the other in the plane $y = t$. The length L of the sheets extends along the z axis.

A current I passes through the top sheet and returns through the bottom sheet. The top conductor has current I directed along the z axis and uniformly distributed along the width W of the sheet. The bottom conductor has the opposite sheet current density

There is a potential difference V between the top and bottom sheet.

- Determine the expressions for the electric and magnetic vector fields in the region between the sheets. (Assume t is small so you can ignore the edge effects. Assume that the fields outside the sheets are zero).
- A non-relativistic beam of charged particles, moving in the z direction with kinetic energy ϵ , passes between the conductors. V and I are adjusted so there is no deflection of this beam from the fields. Show how this device can be used to determine the mass of the beam particles.

Problem 7

Consider a particle of mass m moving in a harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2 X^2$ in one dimension. Let $|n\rangle$ denote the stationary states of the system satisfying

$$H|n\rangle = (n + \frac{1}{2})\hbar\omega |n\rangle$$

Where $n=0,1,2,3,\dots$. Assume that at time $t=0$ the particle is in the state $|\mathbf{j}(0)\rangle$ given by

$$|\mathbf{j}(0)\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{2} |1\rangle + \frac{1}{2} |3\rangle$$

- (a) If we measure the total energy of the system at $t=0$, what possible values will we measure and with what probabilities? How will the respective probabilities change with time?
- (b) What is the state of the system at $t>0$?
What is the mean value of the energy at $t>0$?
- (c) Calculate the mean values of the kinetic and potential energies at $t>0$.

Problem 8

A simplified model for the specific heat of a solid is obtained by considering the N atoms of the solid as independent, isotropic harmonic oscillators of frequency ω , i.e., the solid is a collection of $3N$ simple harmonic one-dimensional oscillators, each with energy levels $E_n = \hbar\omega (n + \frac{1}{2})$, $n = 0, 1, 2, 3, \dots$

- (a) Derive the partition function Z_1 for a single 1D oscillator as a function of $\beta = 1/kT$.
- (b) Determine the mean energy e per oscillator as a function of β .
- (c) Determine the specific heat per atom $C_v(T)$ of the solid.
- (d) Find the high temperature limit of C_v .