Physics Candidacy Exam, P.S.U.
morning, Jan. 12, 2002

Instructions:

The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and the approximations required to arrive at your solution. If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate. Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.

Please use a new blue book for each question. Remember to write your name and the problem number on the cover of each book.

We suggest you read all the questions before beginning to work. You should reserve time to attempt every problem.

<u>Fundamental Constants</u>

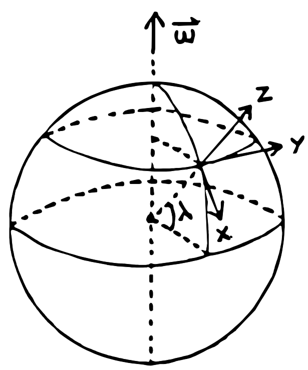
Avagadro's number	N_A	6.02 x 10^{23} (g-mol)$^{-1}$
Boltzman's constant	k_B	1.38 x 10^{-23} J/K
Electron charge	e	1.60 x 10^{-19} C
Planck's constant	h	6.63 x10^{-34} J-s
Speed of light	c	3.00 x 10^{8} m/s
Permittivity constant	ε_0	8.85 x 10^{-12} F/m
Permeability constant	μ_0	1.26 x 10^{-6} H/m
Gravitational constant	G	6.67 x 10—11 N-m^2/s
Electron mass	m_e	9.1 x 10^{-31} kg
Proton mass	m_p	1.67 x 10^{-27} kg

1. (3 points) A particle is projected vertically upward to a height h above a point on the Earth's surface at a northern latitude λ.

a. In which direction is it displaced when it strikes the ground?

b. Show that its displacement is $\frac{4}{3}\omega(\cos\lambda)\sqrt{\frac{8h^3}{g}}$.

(Neglect air resistance and consider only small vertical heights h. The $\vec{\omega}$ is the angular velocity of Earth's rotation around its axis and the Coriolis force is given by $F_{Coriolis} = -2m\vec{\omega} \times \vec{v_r}$.)



2. (3 points) A beam of π^+ particles is created at a well localized source in an accelerator with an energy of 1400 MeV. The lifetime of π^+ is $2.6\text{x}10^{-8}$ s, and its mass is 140 MeV.

a. What is the speed of the particles?

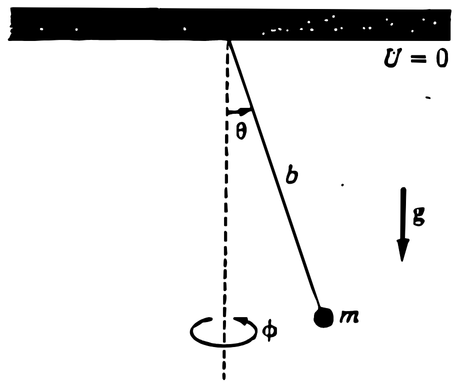
b. At what distance from the source are only 10% of the π^+ particles left?

3. (3 points) Consider a spherical pendulum of mass m and length b.

a. Write down its Hamiltonian.

b. Write down the Hamilton's equations of motion for the pendulum.

c. What are the two conserved quantities in this problem?



4. (4 points) A particle of mass M bounces elastically in one dimension between 2 infinite walls separated by a distance X. The particle is in its lowest possible energy state.

a. What is the energy of the ground state?

b. The separation between the walls is increased (adiabatically) to 2X. How does the expectation value of the energy change?

c. Calculate the energy lost by a ball bouncing off the walls classically during this expansion?
[Hint: We can assume one wall remains fixed.]

5. (4 points) Consider a one-dimensional harmonic oscillator.

a. Write down the Schroedinger equation for the potential $V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$.

b. Show that the trial resolution of the form $\psi(x) = \exp(-\alpha x^2)$ does indeed satisfy the equation provided that

$$\alpha = \frac{m\omega}{2\hbar} \text{ and } E = \frac{1}{2}\hbar\omega$$

c. Show that the product of $\langle p \rangle = \sqrt{E2m}$ and $\langle x \rangle$ is $\langle p \rangle \langle x \rangle \cong \hbar$, where $\langle x \rangle$ is the value of $|x|$ which gives $|\psi|^2 = e^{-1}$

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Welcome back. We hope you enjoyed the pizza to the maximum extent possible under the circumstances.

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6. (4 points) The thermodynamics of a classical paramagnetic system are expressed by the variables: magnetization M, magnetic field B, and absolute temperature T.

The equation of state is $M = CB/T$, where C = Curie constant.
The system's internal energy is $U = -MB$.
The increment of work done by the system upon the external environment is $dW = MdB$.

a. Write an expression for the heat input, dQ, to the system in terms of thermodynamic variables M and B:

$$dQ = (\qquad)dM + (\qquad)dB.$$

b. Find an expression for the differential of the system entropy:

$$dS = (\qquad)dM + (\qquad)dB.$$

c. Derive an expression for the entropy:

$$S =$$

7. (4 points) State which statistics (classical Maxwell-Boltzmann; Fermi-Dirac; or Bose-Einstein) would be appropriate in these problems and explain why (semi-quantitatively):

a. He^4 gas at room temperature and pressure. (Atmospheric pressure is 10^5 N/m^2)

b. Electrons in copper at room temperature. (The density of Cu is 9000 kg/m^3, and its atomic weight is 63.55.)

c. Electrons and holes in undoped semiconducting Ge at room temperature (Ge band-gap $\approx$ 1 volt).

8. (4 points) A conducting object is initially charged with charge q. The object is floating, submerged in a bath of liquid. The conductivity of the liquid is σ and the liquid has dielectric constant κ. The current density in the fluid is then proportional to the electric field.

$$\vec{J} = \sigma \vec{E}$$

a. Write down Gauss's Law and relate it to the surface integral of the current density on a surface containing the object.

b. Write down the relation between the surface integral of current density and the charge on the object (a differential equation for q(t)).

c. How long does it take for 90% of the charge to leak away?

d. If the capacitance of the object is C, determine an expression for the resistance "R" between the object and a grounded surface at infinity. Express R in terms of C, κ and σ.

(Hint: compare the time dependence of this system to a capacitor discharging through a single resistor).

9. (4 points) A neutral object has an electric dipole moment of "p" pointing in the z direction. The molecule is located at x=0, y=0, z=a.

a. Determine the electric field vector at the origin ({x,y,z}={0,0,0}) due to the presence of the dipole molecule.

b. Suppose there is an external electric field in this region. What is the force on the dipole if $\vec{E}(x,y,z) = E_0\hat{z}$, a constant field in the z direction?

c. Find the force on the dipole if $\vec{E}(x,y,z) = \dfrac{E_0\hat{z}}{z^3}$.

Now we insert an infinite conducting plane at z=0.

d. Describe an equivalent image charge configuration that satisfies the boundary condition.

Hint: The electric potential near an isolated dipole is $V(\vec{r}) = \dfrac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$ in SI units.