

1. A simple pendulum of length  $b$  and with a bob of mass  $m$  is attached to a massless support moving vertically upward with constant acceleration  $a$ . Determine:

- a) the equations of motion
- b) the period of small oscillations.

2. The potential for a particle of mass  $m$  moving in one dimension is

$$\begin{aligned} V(x) &= +\infty & (x < 0) \\ V(x) &= 0 & (0 < x < L) \\ V(x) &= V_0 & (L < x < 2L) \\ V(x) &= +\infty & (x > 2L) \end{aligned}$$

Assume the energy of the particle is in the range  $0 < E < V_0$ . Find the energy eigenfunctions and the equation that determines the energy eigenvalues. You need not normalize the eigenfunctions.

3. State the three laws of thermodynamics. Analyze the following process in light of these laws. There is a rigid, thermally isolated container, which is divided into two compartments (of volumes  $V_1$  and  $V_2$ ) separated by a valve which is initially closed. One compartment has an idea gas, while the other is empty. The valve is opened.

- i) Why does the gas expand to fill the entire volume?
- ii) What does the first law tell us?
- iii) Suppose it is found that the temperature of the final state is the same as that before the valve was opened. What does that tell us about the energy?
- iv) How would the results change if instead we consider an adiabatic process in which the volume of an ideal gas changed from  $V_1$  to  $V_2$ ?

4. A block of mass  $m_1$  is at rest on a long frictionless table, one end of which terminates at a wall. A block of mass  $m_2$  is placed between the first block and the wall and set in motion away from the wall and toward  $m_1$  with constant speed  $v_{2i}$ . Assuming all collisions are elastic, find the value of  $m_2$  (in terms of  $m_1$ ) for which both blocks move with the same velocity after  $m_2$  has collided once with  $m_1$  and once with the wall. Assume the wall has infinite mass.

5. A charged particle of mass  $m$  and charge  $q$  moves in a uniform magnetic field  $B\hat{z}$ . Show that the momentum parallel to the direction of the magnetic field is constant and, hence, that the most general motion of the particle is a helix, whose projection onto the  $xy$  plane is a circular orbit with constant angular frequency,  $\omega$ . Find  $\omega$ .

6. Two gravitating masses  $m_1$  and  $m_2$  are separated by a distance  $r_0$  and released from rest.

- a) What is the the magnitude of the velocity of each mass as a function of their separation after release?
- b) Suppose that the masses are given initial velocities at the time of release, but that the center-of mass of the two remains at rest. What condition does this impose on the velocities?
- c) Describe the behavior of the distance of closest approach of the two bodies as a function of the initial velocity vectors meeting the condition in part (b).

7. A plane wave of monochromatic light is normally incident on a uniform thin film of oil that covers a glass plate. The wavelength of the light source can be varied continuously. The index of refraction of the glass is 1.5, and the index of refraction of the oil is 1.3. Minima of the intensity of the reflected light are observed at 500 nm and at 700 nm and at no other wavelengths in between. What is the thickness of the oil film?

8. Consider the earth's atmosphere as an ideal monatomic gas of molecular weight  $\mu$ . Assume that the acceleration due to gravity,  $g$ , is constant with altitude.

- a) If  $h$  denotes the height above the sea level, show that the change of atmospheric pressure  $P$  with height is given by

$$\frac{dP}{P} = -\frac{\mu g}{RT} dh$$

where  $T$  is the absolute temperature at height  $h$ . (Hint: consider a thin horizontal volume element of thickness  $dh$ .)

- b) Assuming  $T$  is the same everywhere (isothermal atmosphere), find the formula for the pressure as a function of the height.
- c) If the decrease of pressure in (a) is due to adiabatic expansion, show that

$$\frac{dP}{P} = \frac{5}{2} \frac{dT}{T}$$

(Remember the gas is monoatomic.) How does the pressure vary with height in this case?

9. Consider two particles that attract each other with the potential

$$V(r) = -\frac{\hbar^2 K^2}{2\mu r^2}$$

where  $r$  is the distance between particles,  $K$  is a constant, and  $\mu$  is the reduced mass. In states of definite angular momentum, for what values of the angular momentum quantum number does the effective force between the particles become repulsive?

10. A cylindrical object of cross-sectional area  $A$  and length  $L$  is made of material with conductivity  $\sigma$ . A conductive electrode is placed on each circular end, and a constant potential difference  $V$  is applied between the two circular electrodes.

- i) Determine the electric field inside the cylinder.
- ii) Calculate the current inside the cylinder.
- iii) Determine the magnetic field near the cylindrical surface. Neglect edge effects.

11. A one-dimensional potential well of width  $L$  and depth  $-V_0$  is placed at the origin. The potential elsewhere is zero. A particle of mass  $m$  and energy  $E$  impinges on the well. Determine the probability that the particle is reflected by the well.

12. Consider a system consisting of two particles, each of which can be in any one of three quantum states of respective energies  $0$ ,  $E$ , and  $3E$ . The system is in contact with a heat reservoir at temperature  $T$ . Calculate the average energy and the specific heat assuming that the particles obey (a) Maxwell-Boltzmann statistics (considered distinguishable); (b) Fermi statistics, and (c) Bose statistics.