Candidacy Exam
Department of Physics
August 25, 2003

Part I

Instructions:

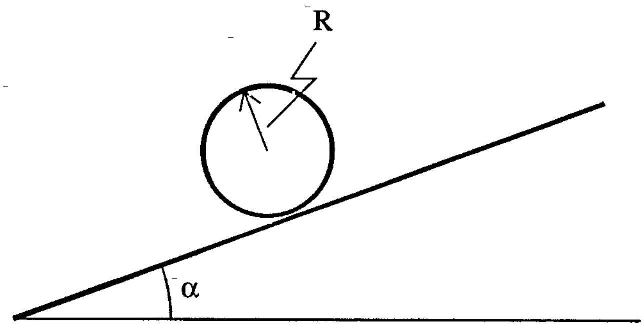
- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and the approximations required to arrive at your solution. If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate. Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all the *four* problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants:

Avogadro's number	N_A	$6.023 \times 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A}^{-2}$
Gravitational constant	G	$6.673 \times 10^{-11}\,\mathrm{m^3\,s^{-2}\,kg^{-1}}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV\,c^{-2}}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV\,c^{-2}}$

I–1 (a) A solid cylinder of uniform density has mass m and radius R. Obtain the moment of inertia of the cylinder about its axis, starting from the definition of moment of inertia.

(b) This cylinder rolls down a fixed inclined plane without slipping. The plane is at angle α above the horizontal. Find the normal force N and the tangential frictional force F acting on the cylinder.

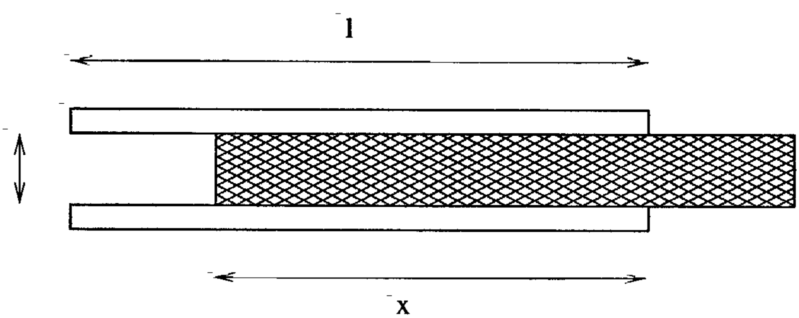


I–2 (a) A capacitor is formed by two parallel square plates of side l separated by a air gap of thickness t. Show that the capacitance is $\epsilon_0 l^2/t$. Throughout this problem, ignore any fringe field effects.

In the following, a constant potential difference V is maintained between the plates by a battery.

(b) A square dielectric slab of relative permittivity ϵ_R is inserted so as to completely fill the air gap. What happens to the energy in the system of battery and capacitor?

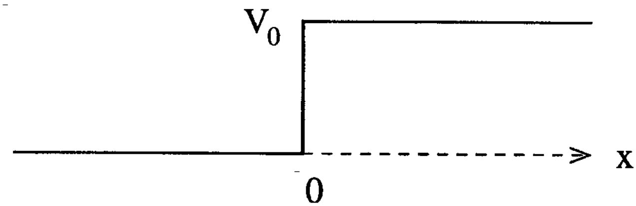
(c) Suppose now that the dielectric slab is slowly removed by withdrawing it parallel to the plates, with the battery still connected. Derive an expression for the magnitude and the direction of the external force that must act on the dielectric slab in the displaced position.



I–3 In one dimension, a nonrelativistic quantum-mechanical particle of mass m and energy $E > V_0$ approaches a barrier from the left. The barrier is a step of height V_0 and is at $x = 0$ — see the figure.

(a) Find the amplitudes of the reflected and transmitted wave functions, given a standard normalization for the incoming wave function.

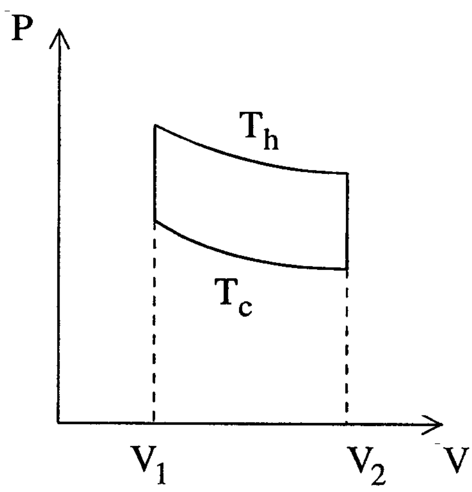
(b) Calculate the current density of particles for $x < 0$ and for $x > 0$.



I–4 A Stirling cycle for an ideal gas is similar to a Carnot cycle except that the two adiabatic legs are replaced by two constant volume legs (see figure). Two legs are at constant volume V_1, V_2 and and the other two are at constant temperatures T_h, T_c.

(a) Derive the change in: (i) internal energy ΔU, (ii) heat Q, and (iii) work W, for each of four legs of the cycle.

(b) What is the efficiency for converting heat from the hot source to work?



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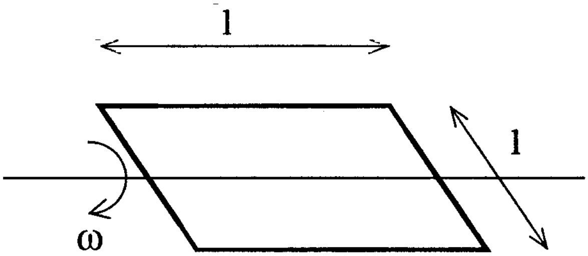
Three identical stars of mass M are located at the vertices of an equilateral triangle with side L. At what speed must they move if they all revolve under the influence of one another's gravity in a circular orbit circumscribing the triangle while still preserving the equilateral triangle?

A square loop of thin copper wire freely rotates about an axis that bisects the loop in the plane of the loop. Perpendicular to the axis is a uniform magnetic field B_0. (See figure.)

How does the rate of dissipation of energy depend on B_0, ω, l, A, ρ, and σ? Here ω, l, A, ρ, and σ are the instantaneous angular frequency, the length of a side of the loop, the cross sectional area of the wire, its density, and its conductivity. It is sufficient to get the power-law dependence correct without deriving the coefficient. You should assume that the conditions are such that only a small fraction of the kinetic energy of the loop is dissipated per rotation, so that a time-average of the dissipation is sufficient for the calculation. Moreover assume that the energy predominantly goes into Joule heating.

Show that ω decays exponentially with a rate independent of l and A.



II–3 A particle of mass m in one dimension is confined to a box $(-L < x < L)$. The potential is zero inside the box and infinite outside.

Write down the energy eigenfunctions and eigenvalues for the three states of lowest energy. Normalize the eigenfunctions.

II–4 Consider a black sphere of radius R at temperature T which radiates to distant black surroundings at zero temperature (0 K). Now surround the sphere with a thin heat shield of radius R_1 that is in the form of a black sphere whose temperature is determined by radiative equilibrium. What is the temperature of the shield and what is the effect of the shield on the total power radiated to the surroundings? [Assume that the heat shield conducts heat perfectly.]