

Graduate Candidacy Examination I
(Sat Aug 25, 2001. 10am-12noon.)

Note: Please complete all FOUR questions. Estimate a maximum of 30mins. for each question. Partial credit will be given for correct verbal reasoning, relevant figures or graphical plots, physically motivated estimates, and any order of magnitude computations.

1. An atomic electron of charge $-e$ circles about the nucleus of charge $+Q$ in an orbit of radius r . The centripetal acceleration is provided by the electrostatic force. Suppose a small magnetic field dB is turned on perpendicular to the plane of the orbit. Show that the increase in kinetic energy, dK , imparted by the induced electric field, is just right to sustain circular motion at the same radius, r as before.

2. A particle of mass m moves in a closed orbit under the influence of the central force $\mathbf{F}(\mathbf{r}) = -\frac{k}{r^2}\hat{r}$.

(a) What is the shape of the orbit? Obtain expressions for the velocity, kinetic energy, and potential energy of the particle in terms of k and m .

(b) Write down Hamilton's equations of motion for the particle in plane polar coordinates. You may use the relations: $\mathbf{r} = r\hat{e}_r$ and $\mathbf{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$, for the position and velocity vectors, respectively.

(c) Show that the angular momentum and areal velocity, $r^2\dot{\theta}$, is conserved. *Hint:* Begin with the \hat{e}_θ eqn of motion obtained in part (b).

3. An atomic particle of mass m is confined to a narrow tube of length L which is closed at both ends.

(a) Find an expression for the wavelength of de Broglie waves which will resonate in the closed tube.

(b) For an electron of mass 9.1×10^{-31} kg, what is the ground state energy for a tube of length $L = 0.5$ nm? You may require Planck's constant: $h = 6.63 \times 10^{-34}$ Js, and the conversion factor: $1\text{eV} = 1.602 \times 10^{-19}$ J.

4. For the central force, $\mathbf{F}(\mathbf{r}) = -\frac{k}{r^n}\hat{e}_r$, we have the orbit equation:

$$\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} = -\mu \frac{r^2}{l^2} f(r) \quad (1)$$

Given that the particle moves in a circle of radius a which can be parameterized as: $r = 2a \cos(\theta)$, show that the power $n=5$, and obtain k in terms of the radius of the orbit, a , the angular momentum, l , and the effective mass, μ .

Graduate Candidacy Examination II
(Sat Aug 25, 2001. 2pm-4pm.)

Note: Please complete all FOUR questions. Estimate a maximum of 30mins. for each question. Partial credit will be given for correct verbal reasoning, relevant figures or graphical plots, physically motivated estimates, and any order of magnitude computations.

1. An idealization of a real crystal is the Einstein crystal which assumes that all $3N$ equivalent harmonic oscillators vibrate with the same frequency ν . The canonical partition function of the Einstein crystal is of the form:

$$Z = \frac{e^{-\frac{h\nu}{2kT}}}{1 - e^{-\frac{h\nu}{kT}}} \quad (1)$$

where k is Boltzmann's constant, and T is the equilibrium temperature.

(a) Find the Helmholtz free energy, $F = -kT \ln Z$, and entropy, $S = -\left(\frac{\partial F}{\partial T}\right)_V$, of the Einstein crystal.

(b) Use the thermodynamic relation, $F = U - TS$, to obtain an expression for the average vibrational energy of an oscillator, $\langle \epsilon \rangle = U/3N$.

2. In an infinite square well of length a there are 5×10^9 electrons per meter. If all of the lowest energy levels are filled, with $E_n = n^2 h^2 / 8ma^2$, determine the energy of the most energetic electron to 3 significant digits. *Hint:* You may find the following constants helpful: $hc = 12.41 \times 10^3 \text{ eV} \cdot \text{Å}$, $m_e c^2 = 0.511 \times 10^6 \text{ eV}$.

3. A 100 Ohm resistor is held at a constant temperature of 300°K . A current of 10A is passed through the resistor for 300s.

(a) What is the change in the entropy of the resistor?

(b) What is the change in the entropy of the Universe?

(c) What is the change in the internal energy of the Universe?

(d) What is the change in the Helmholtz free energy of the Universe?

4. Write down the Maxwell's equations for electrodynamics in free space, assuming charge density, ρ , and current density, \mathbf{J} . Recall that a perfect conductor is a conductor with infinite conductivity; a superconductor is a perfect conductor with the additional property that the constant \mathbf{B} field inside it is zero.

Show that: (i) the magnetic field inside a perfect conductor is constant in time. (ii) the magnetic flux through a perfectly conducting loop is constant. (iii) the current in a superconductor is confined to the surface.